Exercises to Group Theory for Physicists — Sheet 3

Prof. S. Dittmaier and Dr. P. Maierhöfer, Universität Freiburg, SS19

Exercise 3.1   *The dihedral groups*   (4 points)

Enlarge the symmetry group of Exercise 2.2 by a reflection that reverses the x_2 axis, leaving the x_1 axis invariant. This construction defines a two-dimensional representation of the *dihedral group D_n*.

a) Determine all group elements of D_n in the two-dimensional representation given above. What is the order of D_n?

b) Show that the given two-dimensional representation of D_n is irreducible.

c) D_n has two one-dimensional inequivalent representations if n is odd and four one-dimensional inequivalent representations if n is even. Determine these representations.

Exercise 3.2   *Character table of the dihedral group D_4*   (2 points)

The dihedral group D_4 consists of 8 elements generated by two elements ρ, σ with $\rho^4 = \sigma^2 = e$.

a) Construct the character table of D_4.

   Hint: Follow the procedure used in the lecture for the quaternionic group.

b) Is the group uniquely characterised by its character table? Justify your answer.

Exercise 3.3   *Klein's four-group*   (2 points)

Klein's four-group V is the symmetry group of a non-square rectangle. It is defined by the presentation $\langle a, b | a^2 = b^2 = (ab)^2 = e \rangle$.

a) How many irreducible representations does the group have and what are their dimensions?

b) Determine the irreducible representations explicitly. Which ones are faithful?

Please turn over!

Exercise 3.4 *Symmetry group of the ozone molecule*   (2 points)

Consider an electron in the field of three point particles carrying equal positive electric charge that are positioned at the vertices of an equilateral triangle.

a) What is the symmetry group of the Hamiltonian for the electron states? What kind of degeneracy can be expected for energy eigenstates (ignoring possible accidental degeneracies)?

b) What happens to the degenerate energy eigenstates if a homogeneous electric or magnetic field is applied perpendicular to the triangle spanned by the three positive charges?