Exercise 9.1 (6 points) *Vertex functions of scalar QED*

The electromagnetic interaction of spin-0 particles $S^{\pm}$ with mass M (field variable ϕ for particles S^- with charge Q) is described by $\mathcal{L}_\phi = (D_\mu\phi)^\dagger(D^\mu\phi) - M^2\phi^\dagger\phi$ with $D_\mu = \partial_\mu + iQeA_\mu$. This yields the Feynman rules of *scalar QED* given on the back side.

a) Calculate the S-boson self-energy $\Sigma^{SS}(p^2)$ in one-loop approximation in Feynman gauge. The self-energy is defined by the two-point vertex function as follows:
$$\Gamma^{S^+S^-}(-p,p) \;=\; i\left[p^2 - M^2 + \Sigma^{SS}(p^2)\right].$$

b) The three-point vertex function
$$\Gamma^{AS^+S^-}_\mu(k,p_+,p_-) \;=\; -iQe\Big[(p_- - p_+)_\mu + \Lambda_\mu(-p_+,p_-)\Big]$$
fulfills the Ward identity
$$k^\mu\Gamma^{AS^+S^-}_\mu(k,p_+,p_-) \;=\; Qe\left[\Gamma^{S^+S^-}(-p_-,p_-) - \Gamma^{S^+S^-}(p_+,-p_+)\right].$$
Using this identity and the fact that the UV-divergent part of $\Gamma^{AS^+S^-}_\mu$ is proportional to the lowest order $\Gamma^{AS^+S^-}_{0,\mu}$, derive the UV-divergent one-loop contribution to the correction $\Lambda_\mu(-p_+,p_-)$.

c) Analogous to QED with Dirac fermions, calculate the photon self-energy $\Sigma^{AA}_{\mu\nu}(k)$ in one-loop approximation.

Exercise 9.2 (4 points) *Renormalization of scalar QED*

a) Apply the renormalization transformation
$$e \to e(1+\delta Z_e), \quad M^2 \to M^2 + \delta M^2, \quad A^\mu \to A^\mu\Big(1+\tfrac{1}{2}\delta Z_A\Big), \quad \phi \to \phi\Big(1+\tfrac{1}{2}\delta Z_\phi\Big)$$
in one-loop approximation to the Lagrangian $\mathcal{L}_\phi$ given in Exercise 9.1 and determine the generated counterterms (solution on the back side). Which counterterm contributions result for the renormalized vertex functions $\hat\Gamma^{S^+S^-}(-p,p)$, $\hat\Gamma^{AS^+S^-}_\mu(k,p_+,p_-)$ and $\hat\Gamma^{AAS^+S^-}_{\mu\nu}(k_1,k_2,p_+,p_-)$?

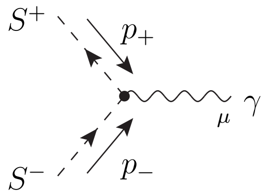
b) Determine δM^2 from the mass-renormalization condition
$$\hat\Gamma^{S^+S^-}(-p,p)\Big|_{p^2=M^2} = 0.$$

c) Determine δZ_ϕ from the renormalization condition
$$\mathrm{Res}_{p^2=M^2}\left\{\hat G^{S^+S^-}(-p,p)\right\} = i$$
for the residue of the S propagator.

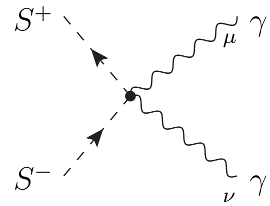
d) Determine δZ_e from the charge-renormalization condition (Thomson limit)
$$\hat\Gamma^{AS^+S^-}_\mu(k=0,-p,p)\Big|_{p^2=M^2} = -2iQep_\mu.$$
Using the Ward identity given in Exercise 9.1 b) show that $\delta Z_e = -\frac{1}{2}\delta Z_A$.

Feynman rules for charged spin-0 particles $S^{\pm}$:

Vertices:

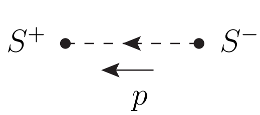


$-iQe(p_- - p_+)_\mu$



$2iQ^2e^2g_{\mu\nu}$

Propagator and external lines:



$$\frac{i}{p^2 - M^2}$$

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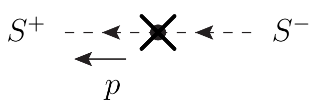
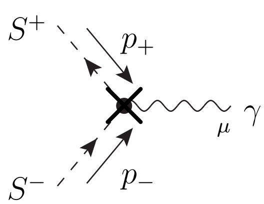
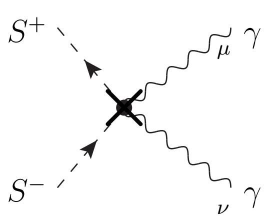
Counterterms:



$i(p^2 - M^2)\delta Z_\phi - i\delta M^2$



$-iQe(p_- - p_+)_\mu\left(\delta Z_e + \delta Z_\phi + \frac{1}{2}\delta Z_A\right)$



$2iQ^2e^2g_{\mu\nu}\left(2\delta Z_e + \delta Z_\phi + \delta Z_A\right)$

The fields $S^{\pm}$ are always assumed to be incoming at the vertex. Outgoing fields $S^{\pm}$ correspond to the incoming fields $S^{\mp}$ with reversed momenta. The Feynman rules for external and internal photons are the same as in QED with Dirac fermions.