

Exercise 10.1 (6 points) *Vertex functions of scalar QCD*

In this exercise the interaction of spin-0 particles S (field variable Φ) in the fundamental representation of $SU(3)$ with gluons will be considered. The scalars are described by the Lagrangian $\mathcal{L}_\phi = (D_\mu \Phi)^\dagger (D^\mu \Phi) - M^2 \Phi^\dagger \Phi$ with the covariant derivative $D_\mu = \partial_\mu + ig_s T^a A_\mu^a$, where $T^a = \lambda^a/2$ are the generators of $SU(3)$.

- a) Write down the Feynman rules of the scalar particle S .
- b) Calculate the self-energy $\Sigma^{SS}(p^2)$ of the S bosons in one-loop approximation in the Feynman gauge. The self-energy is defined via the two-point vertex function as

$$\Gamma^{SS}(-p, p) = i \left[p^2 - M^2 + \Sigma^{SS}(p^2) \right].$$

- c) Calculate the contribution of the scalars to the gluon self-energy $\Sigma_{\mu\nu}^{g_a g_b}(k)$ in one-loop approximation.

Exercise 10.2 (4 points) *Ghost renormalization in QCD*

The renormalized two-point vertex function of the ghost fields in QCD is given by

$$\hat{\Gamma}^{\bar{u}_a u_b}(p, -p) = i \delta^{ab} \left[p^2 + \Sigma^{\bar{u}u}(p^2) + p^2 \delta Z_u \right]$$

with the unrenormalized self-energy $\Sigma^{\bar{u}u}$ and the wave-function renormalization constant δZ_u .

- a) Calculate the self-energy $\Sigma^{\bar{u}u}(p^2)$ of the ghosts in one-loop approximation in the Feynman gauge.
- b) Determine δZ_u in $\overline{\text{MS}}$ renormalization, so that $\hat{\Gamma}^{\bar{u}_a u_b}$ is UV-finite and the renormalization constant is proportional to the constant $\Delta = \frac{2}{4-D} - \gamma_E + \ln(4\pi)$.